where c^s_{11}, c^s_{111}, and c^s_{1111} represent isentropic second-, third-, and fourth-order elastic constants. Unlike conventional experiments which evaluate high-order elastic constants at low strain, the present paper will interpret the coefficients of Eq. (6) as those which define the expansion over the range of elastic compressions achieved in the experiments.

B. Piezoelectric Elastic Solids

The piezoelectric constitutive relation is formulated as for the elastic solid except that the internal energy is further considered to be a function of the electric field E. The internal energy function $\overline{U}=\overline{U}(\eta,\ s,\ E)$ is expanded at constant entropy. When the derivatives of this expansion, $t=\rho_0(\partial\overline{U}/\partial\eta)$ and $D=\rho_0(\partial\overline{U}/\partial E)$, are specialized to uniaxial strain and electric field, the constitutive relations are

$$t_1=c^E_{11}\eta_1+\tfrac{1}{2}c^E_{111}\eta_1^2-e_{11}E_1-\tfrac{1}{2}\frac{\partial e_{11}}{\partial E_1}E_1^2-\frac{\partial e_{11}}{\partial\eta_1}\eta_1E_1+\tfrac{1}{6}c^E_{1111}\eta_1^3+\text{h. o. t.} \quad (7)$$

and

$$D_1=e_{11}\eta_1+\tfrac{1}{2}\frac{\partial e_{11}}{\partial\eta_1}\eta_1^2+\epsilon_1E_1+\tfrac{1}{2}\frac{\partial\epsilon_1}{\partial E_1}E_1^2+\frac{\partial\epsilon_{11}}{\partial\eta_1}\eta_1E_1+\text{h. o. t.}\ , \quad (8)$$

where ϵ_{11} is the permittivity at constant strain.

The present experiments are conducted with the electrodes connected with an effective electrical short circuit. This condition imposes time-variable electric fields in the sample and greatly complicates exact analysis of the data. Fortunately, however, the interaction among the elastic and piezoelectric constants in X-cut quartz is smaller than the experimental errors, and the interactions can be neglected in the evaluation of Eq. (7). Hence, in determining the elastic constitutive constants for X-cut quartz the "weak-coupling" approximation will be employed. This has the effect of interpreting the data as if the piezoelectric contribution to stiffness were zero even though the experiment involves a small contribution to each constant from the piezoelectric stiffening. This contribution is equal to 0.86% of c^E_{11} in the linear theory.[21] Thurston *et al.*[22] have noted that differences between third-order constants in quartz due to different electrical boundary conditions were too small to be detected. Thus, the weak-coupling approximation should provide an excellent approximation to the experiment. To emphasize the approximation, however, the notation specifying the electrical boundary condition is dropped from the elastic constitutive relation.

All the terms in the piezoelectric constitutive relation, Eq. (8), are of potential influence. Carr[5] has estimated a value of $\partial\epsilon_1/\partial E_1=-10^{-22}$ C V^{-2} which will have a negligible contribution of less than 0.1% in ϵ_{11} at the maximum field encountered in the present experiments. Equations (7) and (8) are then reduced to

$$t_1=c_{11}\eta_1+\tfrac{1}{2}c_{111}\eta_1^2+\tfrac{1}{6}c_{1111}\eta_1^3+\text{h. o. t} \quad (9)$$

and

$$D_1=\left(e_{11}+\frac{1}{2}\frac{\partial e_{11}}{\partial\eta_1}\eta_1\right)\eta_1+\left(\epsilon_{11}+\frac{\partial\epsilon_{11}}{\partial\eta_1}\eta_1\right)E_1+\text{h. o. t}\ . \quad (10)$$

The experimental results will be expressed in terms of the nonlinear elastic and piezoelectric constitutive relations given in Eqs. (9) and (10). The analysis will be accomplished in such a way as to determine the applicability of the form of the constitutive relations to the observed large-strain behavior of quartz and to determine the appropriate constants.

Before proceeding with the development of an electrostatic model which relates the experimental observations to the constitutive relations, it will be helpful to describe the experimental configuration.

III. EXPERIMENTAL

The experimental data are obtained from samples in a one-dimensional configuration in which planar elastic shock waves are introduced by precisely controlled planar impacts. As the elastic shock wave imparted by the impact propagates through the sample, the resulting short-circuited current pulse is recorded. These current-vs-time pulses may then be analyzed with an electrostatic model to obtain values for the piezoelectric polarization, the wave speed, and the ratio of strained to unstrained permittivity at each of a number of different strain amplitudes. The strain amplitudes are chosen to have values which range from those in which nonlinear contributions are negligible to those in which nonlinear contributions are substantial.

The planar-impact technique is now widely and routinely employed to measure both electrical[23] and mechanical[24] responses of shock-loaded solids. Considerable improvement in the critical details of the experimental technique have been accomplished since the previous study[11] of the response of shock-loaded quartz. Experimental details were recently described[25]; hence, only the main features of the technique will be described here.

As shown in Fig. 1, X-cut quartz specimen disks are encapsulated in Epoxy potting and attached to the muzzle of a compressed-gas gun.[26] The projectile, faced with an X-cut quartz disk, is accelerated down the evacuated barrel and impacted upon the specimen with precise control on the alignment of the impacting surfaces. Immediately prior

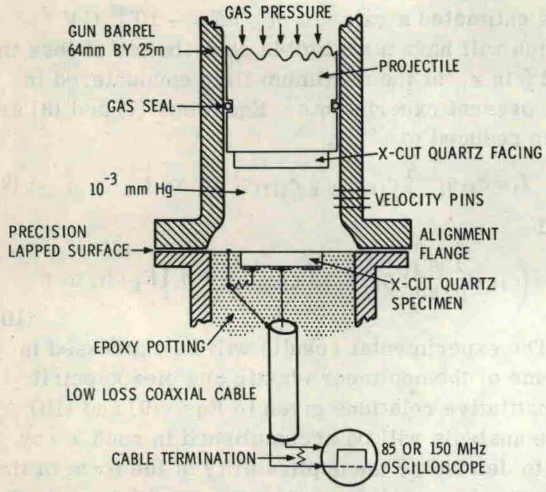

FIG. 1. Elastic shock waves are introduced into X-cut quartz samples by the precisely controlled impact of projectiles faced with X-cut quartz disks. As the elastic shock wave propagates through the sample, the resulting short-circuited piezoelectric current is monitored with a low-impedance resistive circuit connecting the two electrodes. Immediately prior to impact, the velocity of the projectile is measured to an absolute accuracy of $\pm 0.1\%$. Impact velocities of from 27 to 510 m sec^{-1}, corresponding to strains of from 2.4×10^{-3} to 4.3×10^{-2}, were utilized in the present investigation. The sample is constructed with a peripheral guard ring which ensures states of uniaxial strain, uniaxial piezoelectric polarization, and uniaxial electric field along the x axis of the quartz disk.

to impact, the velocity of the projectile is measured by coaxial pins which protrude through the side of the barrel. The short-circuited current from the specimen is monitored with a low-impedance resistive circuit, displayed on a high-frequency oscilloscope, and recorded on high-speed Polaroid film. Thus, the apparatus produces a well-controlled impact between identical materials and provides an accurate measure of the projectile velocity as well as an accurate measure of the current-time pulse from the shock-loaded sample. Various strain values are achieved by experiments at various preselected velocities; the present experiments were conducted from 27 to 510 m sec^{-1}.

Due to the symmetry of the impact of identical materials, the particle velocity u imparted to the specimen is

$$u = \tfrac{1}{2}u_0 ,$$

where u_0 is the velocity of the projectile at impact. The projectile velocity is measured with a maximum error of 0.1%.

The diameters and thicknesses of the specimen and projectile facing disks are chosen to prevent unloading from lateral surfaces during a single wave transit time. This ensures a state of uniaxial strain in the specimen. The thickness of the facing disk is chosen to control the arrival time of the unloading from the rear of the facing. In most cases the unloading time was longer than the shock-wave transit time through the sample. Some of the data were obtained during the previously reported study of the unloading response of X-cut quartz.[13] In those cases the thickness of the facing disk was chosen to cause unloading at preselected times.

The specimen is constructed from a large diameter to thickness disk with a peripheral guard ring applied to the electrode opposite the impact face. It has previously been demonstrated that a configuration in which the width of the guard ring is equal to or greater than 1.5 times the thickness of the disk will ensure states of one-dimensional strain and electric field in the central region of the disk for one wave transit time.[11] The present experiments confirm that observation. However, at the very low impact velocities, i.e., < 80 m sec^{-1}, it was noted that wider guard-ring widths were required to obtain current pulses which agreed with the electrostatic model. This latter condition is apparently due to impacts occurring first at the peripheral area of the disk; the resulting signal from the guard-ringed area is then coupled electrostatically into the inner electrode signal.

The piezoelectric effect produces very large electric fields; (10^4–10^6 V cm^{-1}); hence, considerable care is taken to ensure excellent electrical insulation on the lateral edges of the disk and in the insulating gap cut between the inner electrode and the guard ring. The insulating gap has an area typically 3% of the area of the inner electrode. The effective collecting area of the inner electrode is taken to include one-half the area of the insulating gap.

Each specimen is visually inspected for defects under a strong side light. The material investigated was synthetic quartz grown by Sawyer Research Products and cut to the disk configuration by the Valpey–Fisher Corp. Each experiment is destructive; hence, variation in material properties from sample to sample will introduce statistical errors. Statistical errors due to sample-to-sample variation of properties were observed to be less than the experimental errors.

Planar impact is assured by rigidly controlling all dimensions contributing to the alignment. For the thick impactors used in the present investigation the median deviation from a planar impact, called "tilt," was 250 μrad. Experiments for impact velocities less than 50 m sec^{-1} require exceptional control on alignment. A median value of tilt of 150 μrad was achieved.

The velocity imparted to each specimen is known to a maximum error of 0.1%; hence, the accuracy of the piezoelectric constants is limited by the ab-